Deriving equations for coupling between digital signal lines removes the element of mystery which often surrounds crosstalk; curves for basic constants, together with a table of general waveforms, allow rapid, quantitative prediction in specific systems

Predicting Crosstalk in Digital Systems

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Digital logic circuits currently in use have delay times which compare to those of the packaging interconnections. At high speeds, interconnections no longer behave as simple short circuits, but take on the appearance of transmission lines. Associated with these lines is a phenomenon called interconnection noise, ie, extraneous voltages and currents caused by reflection and crosstalk. Reflection, and the transmission line terminations required to control it, are well documented in recent literature. Crosstalk, however, remains something of a mystery to the digital designer because of the lack of literature on the subject. This has resulted in overdesigned systems and, even worse, in systems which don't work.

The trend in large mainframe circuit selection is toward high speed ECL, although both ECL and Schottky TTL are currently being designed into medium-sized computers. The problem of crosstalk—coupling between transmission lines—is becoming more important to the designer and, in the future, could make the difference between successful and failing systems.

Crosstalk is discussed here at an engineering level, in terms of both its physical and mathematical developments under systems conditions. Emphasis is on printed wire connections since the engineer has direct design responsibility for boards.

Physical Characterization

Consider the situation shown in Fig. 1, where two parallel transmission lines are in such close proximity that appreciable mutual capacitance (C_M) and mutual inductance (L_M) exist between the lines. (Initially, only the incremental effects at point X will be considered. Assume that both lines are identical with regard to their characteristic impedance, Z_0, and their total delay, T; both are terminated in this impedance. The active or driver line transmits a pulse waveform, V(t). The second line should not be carrying a pulse, and for this reason it is called the passive line. When the pulse arrives at point X, a capacitive current results on the passive (or sense) line. Since mutual capacitance (C_M) sees equal capacitance looking either way on the passive line, equal and opposite currents, i_C, are developed on the passive line. Each current travels toward one end of the line. At the same time, due to mutual inductance (L_M), an equal but opposite current, i_L, is developed in the passive line (Lenz's Law).

At point B, currents i_C and i_L are additive and in such a direction so as to produce a voltage drop with the same polarity as the source voltage of the active line. At point F on the passive line, i_L and i_C are in opposition. If both lines are contained in a completely homogeneous medium, i_L and i_C, at point F, cancel exactly and no waveform results. If a nonhomogeneous medium is present, i_L is normally greater than i_C. In this case the currents are subtractive and produce a voltage drop with opposite polarity of the active line source voltage.

Now consider propagation of the pulse source voltage along the active line. This pulse is of amplitude V, with a rise time of a. Its width is assumed to be much longer than delay T of the two lines in question. At the instant when V(t) is at point (1) on the active line, two pulses are induced in the passive line, one

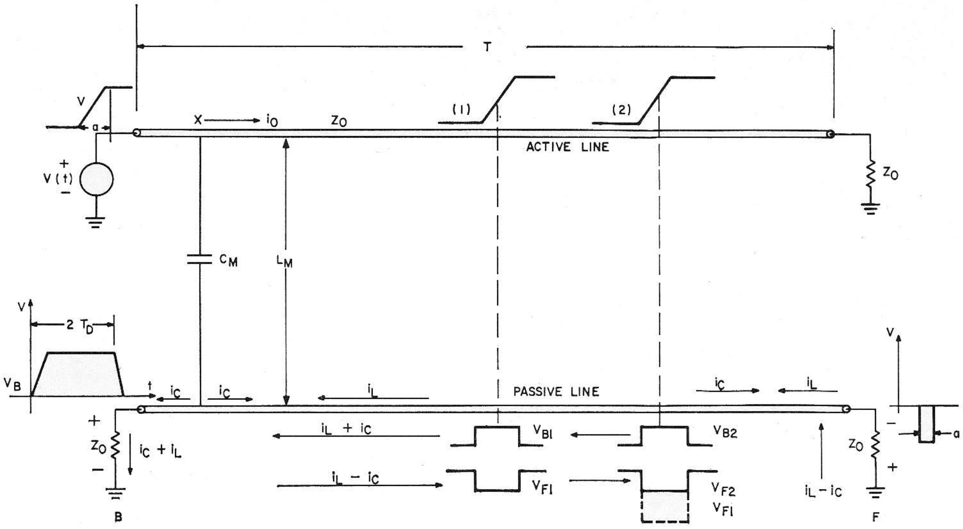

Fig. 1 Crosstalk between two parallel lines. Physical basis for crosstalk is seen to be the adding and subtracting of induced capacitive and inductive currents

of which, V_{B1}, is produced by ($i_C + i_L$) and will propagate toward terminal B. The other, V_{F1}, is produced by ($i_L - i_C$) and propagates toward terminal F. At a later time, V(t) moves to position (2) on the active line. Again V(t) induces two pulses, V_{B2} and V_{F2}, which are identical to those induced at position (1).

During the time it takes for V(t) to go from position (1) to position (2), V_{F1}, traveling at the same velocity and in the same direction (forward), arrives at the point on the passive line where V_{F2} is being induced. V_{F1} and V_{F2} exist at the same time and add directly. As V(t) travels along the passive line it continually induces a series of pulses, V_F, traveling forward. Each new pulse is produced at the exact instant that all previously generated pulses have arrived at this point. The pulses continue to add directly as V(t) travels down the line. If the lines are coupled for a greater distance, a larger number of pulses is produced and the signal amplitude is correspondingly greater. At the end of the line (point F) there is a waveform whose amplitude is proportional to the length of the line and in opposite polarity to signal voltage V(t). Since this waveform is produced by pulses traveling in the forward direction of the signal, it is called the forward crosstalk voltage.

While V(t) travels along the active line producing V_F pulses, it causes the previously mentioned V_B pulses. As V(t) propagates down the line it also continues to generate V_B pulses. It generates the first pulse when V(t) is at the beginning of the line and the last when V(t) reaches the end—a period which took time T. The last V_B pulse, produced when V(t) is at the end of the line, travels backward on the line. It must travel for the same length of time, T, from the end to the beginning of the passive line. While V(t) moves down the line, it generates V_B pulses, which arrive one after the other in a time sequential manner. The first is generated at time $T = 0$; the last arrives at the beginning of the line after time 2T. As a result, the pulse formed by these backward-traveling pulses, or backward crosstalk voltage, has a width equal to twice the line delay. While the amplitude of the forward crosstalk waveform is a direct function of line delay, the backward crosstalk waveform's amplitude is independent of line length (assuming pulse rise time to be less than 2T).

Pulse width of the forward crosstalk voltage can be reasoned to be related to the length of time it takes V(t) to pass every incremental point along the line. In other words, forward crosstalk pulse width is approximately equal to signal voltage risetime.

Basic Crosstalk Waveforms

The basic technique in verifying the previously discussed results is to relate currents and voltages on the pair of lines by a set of differential equations, and then by solving these equations, establish crosstalk waveforms.[1] The section of line to be considered here is shown in Fig. 2, in which the lines are the same as in Fig. 1. $C\Delta X$ is the capacitance of the line having an incremental length, ΔX. This situation holds for self-inductance $L\Delta X$, mutual capacitance $C_M\Delta X$, and mutual inductance $L_M\Delta X$.

Writing loop equations around loops a, b, c, d and e, f, g, h, as well as node equations at points b and f results in the following set of equations—after dividing through by ΔX, eliminating common terms, and taking the limit as ΔX approaches zero.

$$-\frac{\delta V_1}{\delta X} = L\frac{\delta i_1}{\delta t} + L_M\frac{\delta i_2}{\delta t}$$

$$-\frac{\delta V_2}{\delta X} = L\frac{\delta i_2}{\delta t} + L_M\frac{\delta i_1}{\delta t}$$

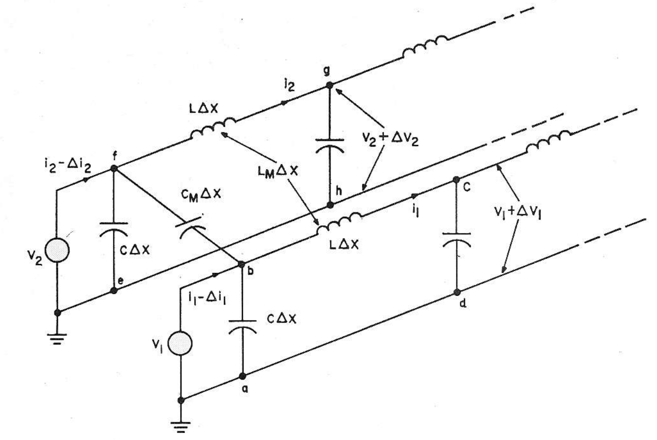

Fig. 2 Incremental length of cross-coupled lines used to write loop and node equations from which crosstalk voltages are derived

$$-\frac{\delta i_1}{\delta X} = (C_M + C)\frac{\delta V_1}{\delta t} - C_M\frac{\delta V_2}{\delta t}$$

$$-\frac{\delta i_2}{\delta X} = (C_M + C)\frac{\delta V_2}{\delta t} - C_M\frac{\delta V_1}{\delta t}$$

By using differential equation theory and Laplace transforms, and assuming that coupling between lines is not very large, we can solve for $V_2(t)$ as a function of X. For both lines terminated in their characteristic impedance, as shown in Fig. 3(a), the solution at the ends of the line is:

at $X = 0$,
backward crosstalk $= V_2(t)$

$$= K_B\frac{V}{a}[tU(t) - (t-a)U(t-a) - (t-D_1)U(t-D_1) + (t-D_2)U(t-D_2)]$$

at $X = l$,
forward crosstalk $= V_2(t)$

$$= K_F\frac{V}{a}[U(t-D_3) - U(t-D_4)]$$

where

$K_B = \frac{\gamma}{4}(K + 1) =$ backward crosstalk constant

$K_F = \frac{\gamma(K-1)}{2u} =$ forward crosstalk constant

$\gamma = \frac{C_M}{C + C_M}$

$u = \sqrt{\frac{1}{L(C + C_M)}}$

$K = \frac{L_M}{\gamma L}$

$a =$ rise time of driving voltage

$l =$ line length

D_1 through $D_4 =$ delays; ie,

$$D_1 = \frac{2l}{u},\ D_2 = \frac{2l}{u} + a,\ D_3 = \frac{l}{u},\ \text{and } D_4 = \frac{l}{u} + a$$

The backward crosstalk waveform has non-zero rise and fall times. Forward crosstalk is a step with a pulse width equal to driving voltage rise time.

Waveforms for Alternate Terminations

The basic equations for $V_2(t)$ describe crosstalk between a terminated active line and a passive line terminated in its characteristic impedance at both ends. In an actual system it is possible that the passive line will not be terminated at both ends. This condition occurs when either a high impedance input circuit or a low impedance output circuit might be loading the line. Since crosstalk amplitudes and waveshapes are affected by these changes in termination, important cases will be considered here.

Continue to assume that a backward and forward crosstalk waveform is induced in the unterminated lines. Naturally these waveforms will undergo reflections. By considering both induced waveforms with respect to lack of terminations, final waveforms can be arrived at. The mathematical derivations consist of solving differential equations subject to boundary conditions determined by the particular termination case under study. Boundary conditions and time responses are shown in Figs. 3(a) through 3(f).

As an example, consider Fig. 3(c), where line 2 has an open circuit at its far end ($X = l$) and is still terminated at its near end ($X = 0$). An open circuit results in a reflection coefficient of 1, which means that the entire waveform is reflected and travels back toward the beginning of the line. This complete reflection results in the waveform doubling at the far end. We then expect to see a negative forward crosstalk waveform twice the amplitude of the previous case. Since the reflected waveform is moving back toward $X = 0$, we expect it to combine with the backward crosstalk wave-

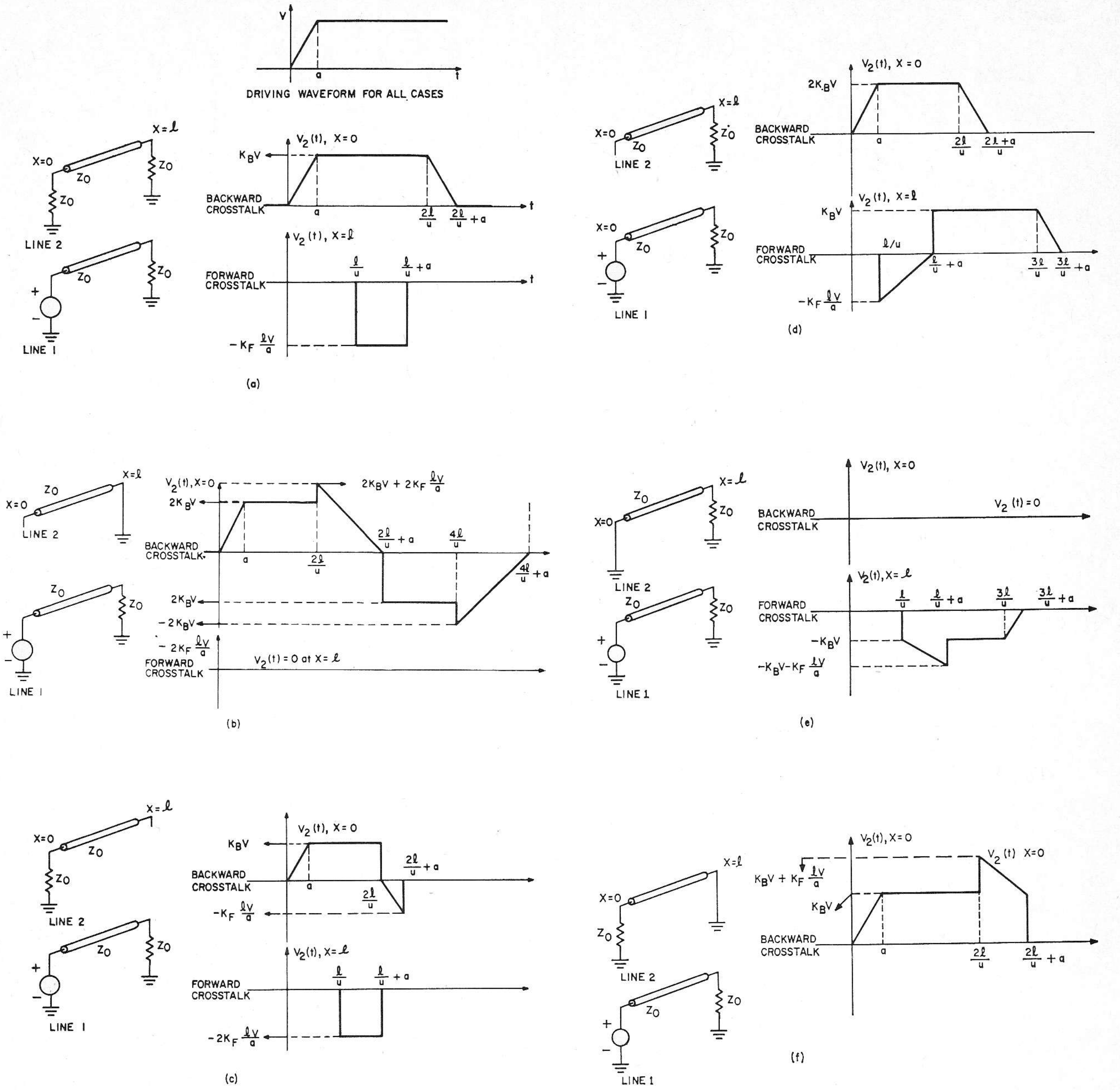

Fig. 3 Forward and backward crosstalk waveforms for common line terminations. By applying parameter values of a specific system to these general diagrams, quantitative predictions of crosstalk waveforms in printed circuit transmission lines can readily be made

form. The passive line sees Z_0 at $X = 0$ and no additional reflections take place. The waveform at $X = 0$ is thus the normal backward crosstalk waveform, plus forward crosstalk appearing as a negative voltage at time 2T. These waveforms are shown in Figure 3(c). Additional cases of terminations can be arrived at using similar reasoning. Results for the most common types are shown in Figs. 3(a) through 3(f).

The previously developed equations and resulting waveforms give a clear picture of relative crosstalk properties of various conditions occurring on coupled transmission lines. So that this information may be used as a quantitative design aid, it is important to obtain numerical values for terms which appear in these equations. V/a, slope of the input rise time, and l, length of the line, are known by the designer; quantities K_B and K_F, the backward and forward crosstalk constants, remain to be found. The fact that, despite the existence of at least six different cases only two distinct terms are present, is extremely fortunate. The determination of only two terms enables us to determine the crosstalk for any of the six.

To properly determine crosstalk constants from capacitances, inductances, impedances, and delays according to the relationships presented earlier, we must be able to measure or calculate C_M and L_M—a difficult task, in general. One method of determining crosstalk constants is by direct measurement of crosstalk under controlled conditions.[2] General enough to be used with any kind of coupled lines, this technique essentially consists of setting up the case of Fig. 3(a) on two lines and applying the forward and backward crosstalk equations. A fast rise time pulse generator and sampling scope should be used.

Crosstalk Constants for Printed Circuits

Today's digital electronics systems generally use printed circuit (PC) boards. This type transmission line is important to the designer not only because of its wide usage, but because it, more than other types of lines, is designed by the circuit engineer rather than purchased. In other words, the engineer is responsible for the electrical and mechanical conditions that may have to be imposed on these lines in order to provide a working system. For the purpose of developing numbers for crosstalk constants we shall distinguish between types of printed wire interconnections. Four types are shown in Fig. 4; in each, a ground plane is present.

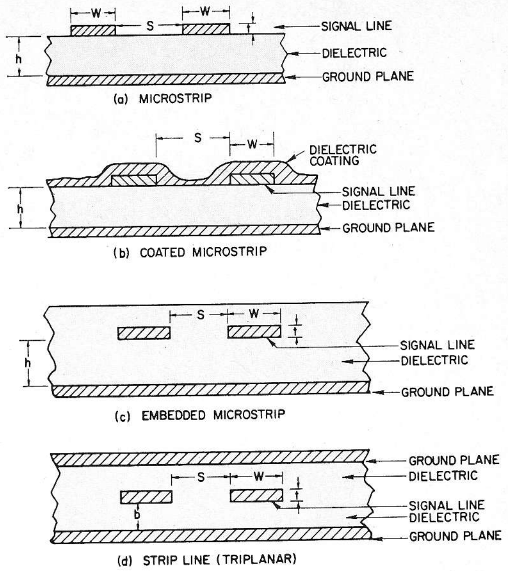

Fig. 4 Cross-sections of the most common types of printed circuits used in digital systems. Conductor width and thickness, distance between conductors, and distance of conductor from ground plane together with dielectric constant determine crosstalk constants

Microstrip Lines

The most common type PC transmission line is the microstrip line, shown in Fig. 4(a). Associated curves (Fig. 5) plot the two crosstalk constants as a function of line spacing, S, with height, h, from the ground plane as a parameter. For the range of practical values, crosstalk constants are independent of line widths. Typical data indicate that for 7.5 mils $\leq W \leq$ 25 mils, crosstalk constants vary only about 0.5 percent with line widths and, as expected, decrease with decreasing line spacing and increase with height from the ground plane. Curves for K_B may be used for values of h not included in the figure. Values may be scaled from one curve to produce another. As an example, consider the K_B curve for h = 44 mils; at S = 44, K_B is approximately 0.08. In other words, for $(S/h) = 1$, $K_B \approx 0.08$. Checking the curves for other values of h, using the fact that S/h reveals approximately the same value of K_B, we see that for h = 59.6 and S = 60, (and thus for h = 15 and S = 15), K_B is approximately 0.08. This can be continued by taking various values of S/h from an existing curve to produce a new one; results are more than accurate enough for most en-

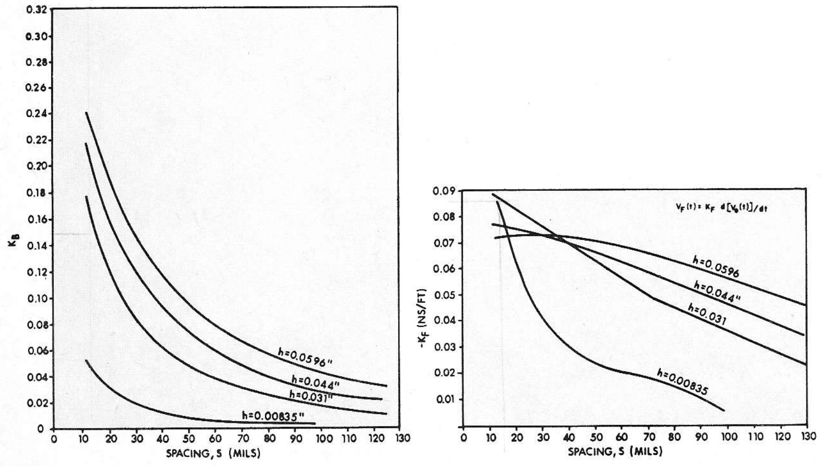

Fig. 5 Empirically derived microstrip line forward and backward crosstalk constants as a function of conductor spacing with separation from ground plane as a parameter. For this derivation, both lines were terminated in their characteristic impedance at both ends, and copper thickness was 2.8 mils. Curves are valid for line widths between 7.5 and 25 mils. Time delay of the line is 1.8 ns/ft

gineering applications. This technique is not applicable for forward crosstalk curves.

The use of Fig. 5 is illustrated by a simple example. Assume that the circuit designer has the configuration shown in Fig. 3(c), where $V = 5$, $a = 2$ ns, $l = 1.25$ ft, $h = 26$ mils, $W = 13$ mils, and $S = 13$ mils. Of interest is determining amplitudes and time positions of the crosstalk waveforms at the ends of line 2. To use the curves, first determine if $a \leq 2T$, where $T = l/u$. (The case where $a > 2T$ will be discussed later.) Since l/u is about 1.8 ns/ft and $l = 1.25$ ft, $2l/u = 2T = 4.4$ ns. Fig. 5 may then be used directly. Using $S = 13$ mils and $h = 26$ mils, we can read $K_B \approx 0.15$ by interpolation from Fig. 5. From Fig. 3(c) we see that the positive amplitude of crosstalk at the input end of the line is $V_{pos} = (5 \text{ x } 0.15) = 0.75$ V. The waveform at the input end will also have a negative component; from Fig. 5, $K_F = 0.085$ ns/ft, and $V_{neg} = -K_F l(V/a) = 0.085 \text{ x } 0.125 \text{ x } 5 = 0.26$ V. The waveform at the output end of the line is $V_{-B} = -2K_F = l(V/a) = -0.53$ V, and is 2 ns wide. Calculated and actual waveforms are shown in Fig. 6. Note that due to the finite rise time of the scope, no step-like edges ever appear. Agreement with the calculated waveforms is quite good.

In Fig. 3(a), if the signal rise time exactly equals the 2-way line delay, ie, $a = (2l/u)$, the backward crosstalk waveform begins to decrease at this point in time ($t = a$). We have seen that if $a \leq (2l/u)$, backward crosstalk is independent of line length, but when $a > (2l/u)$, backward crosstalk is a function of the relationship between a and $2l/u$. The exact relationship between signal rise time a and line delay to determine a specific value of K_B can again be seen from Fig. 3(a), where the amplitude actually reached is proportional to $2l/ua$. If $a = 3(2l/u)$, actual K_B is $K_B/3$, or $K_B(a > 2T_D) = K_B(2(l/u))/a$. (Forward crosstalk constant K_F is, as previously explained, always a function of a and T.) Implications of this equation are by no means obvious. If for mechanical reasons S and h are fixed quantities and the crosstalk level is too high, it may be reduced to a tolerable level by changing either a or l. It is usually easier to change l because of the lack of predictability in altering circuit risetime a.

To illustrate this technique, again consider the previous case. Assume that the 0.75-V crosstalk level is too high and that to control systems noise it should be reduced to 0.5 V. This means that for a 5-V signal, line length will have to be reduced below $2l/u$;

$$\therefore K_B\ (a > 2T) = \frac{0.5 \text{ V}}{5 \text{ V}} = 0.1$$

and

$$l = \frac{aK_B(a > 2T)}{(1/u)2K_B} = \frac{2 \text{ x } 0.1}{1.8 \text{ x } 2 \text{ x } 0.15} = 0.37 \text{ ft}$$

This is the maximum length required to reduce the crosstalk to 0.5 V.

The graphs and equations presented can be extended to embedded microstrip lines (Fig. 4(c)) if the distance from surface to conductor is approximately 1.5 times conductor thickness. Letting subscript $_E$ identify embedded parameters such as C_E, L_E, etc, the following observations[3] are true:

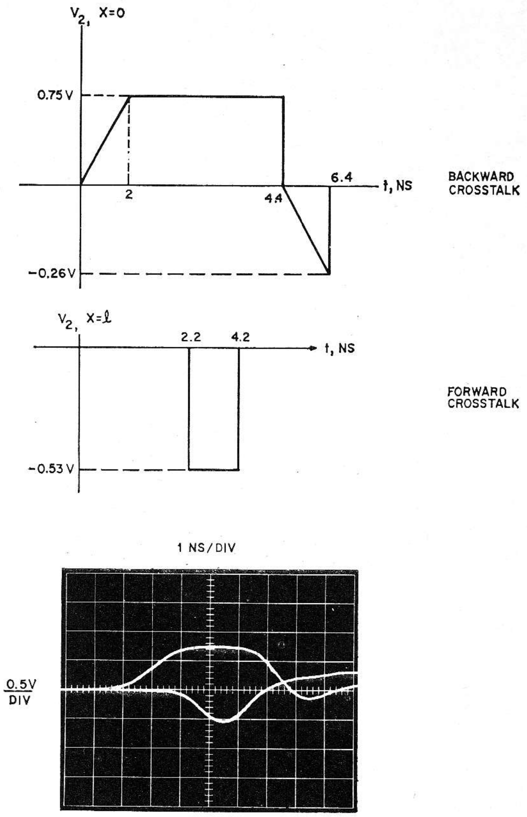

Fig. 6 Calculated and actual crosstalk waveforms for line 2 terminated in its characteristic impedance at X = 0 and open at X = l (Fig. 3(c)). Input voltage is a 5-V ramp with 2-ns rise time

$$T_{DE} = \frac{\sqrt{\epsilon r}}{u} = 2.3 \text{ ns/ft (fiberglass epoxy)}$$

$$Z_{OE} = 0.78\ Z_o$$

$$C_E = 1.64\ C$$

$$L_E = L$$

Arvanistakis[4] has shown that for a homogeneous dielectric no forward crosstalk exists, and $K_{BE} = K_B - (K_F/2T_D)$, where K_{BE} is the crosstalk constant for embedded microstrip lines.

Strip Line (Triplanar)

A curve for finding K_B of a triplanar strip line is shown in Fig. 7. As with the other types of line completely surrounded by dielectric, forward crosstalk is zero. The backward crosstalk voltage of this structure was studied by Catt.[5] Fig. 7, based on his work, gives K_B as a function of S/b. The one curve may be used with reasonable accuracy for the same values of W as before; ie, $7.5 \leq W \leq 25$ (mils).

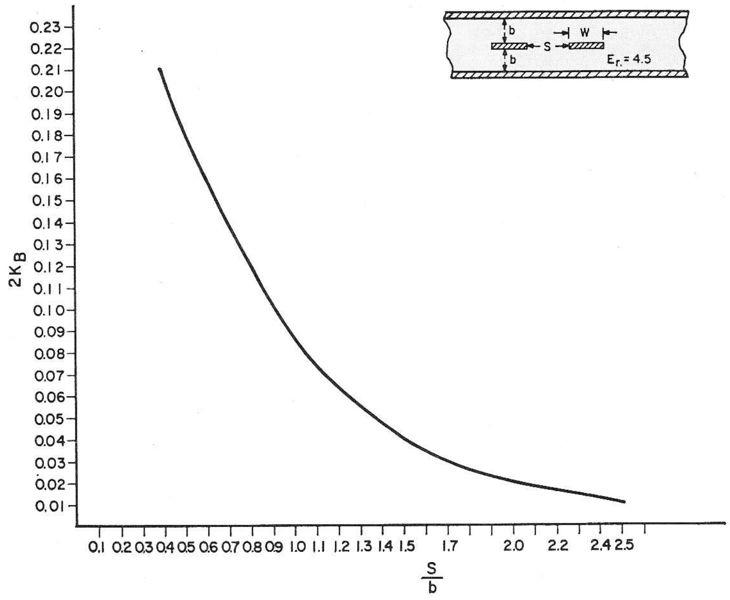

Fig. 7 In the triplanar strip line, only forward crosstalk exists. Forward crosstalk constant value can be determined from the single curve shown here

Other Types of Transmission Lines

Crosstalk can exist between lines of any type. With the common types, the digital design engineer is usually responsible only for design of printed wire boards. Coaxial, flat, and twisted pair cable assemblies are normally purchased from a vendor. Because of problem complexity, the great variety of dielectrics, and geometrical configurations, little analytical work describing general crosstalk properties of any of these transmission lines has been published. However, manufacturers normally provide extensive empirical data concerning their particular products. In general, their approach follows techniques indicated previously. Although it is not possible to provide data here on the vast combinations of materials and geometrical configurations available, it is a simple matter to either consult vendor literature or make one's own measurements using the techniques presented.

Conclusion

At speeds of today's Schottky TTL and ECL, circuit interconnections behave like transmission lines. Associated with this behavior is crosstalk—the generation of noise due to the proximity of switching signal sources. Crosstalk is caused by the mutual capacitance and inductance which exist between parallel transmission lines. These mutual components give rise to inductive and capacitive currents which add at the near end of the line and subtract at the far end. Two crosstalk waveforms result whose shapes may be altered by various conditions of the line termination. A mathematical analysis provides additional insight so that empirical data may be taken to generate tables of crosstalk constants. Such tables can be used as design aids for optimizing PC board interconnections.

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